

Exercise No. 14: The Dirac Equation II
The Dirac Equation in the Electromagnetic Field

1. Find the conserved probability current in the presence of an electromagnetic field and compare with the non-relativistic result.
2. Find the wave-functions and energy eigenvalues of a Dirac particle in a magnetic field along the z -direction.

Hint: Write the quadratic equation, substitute the trial solution

$$\Psi = \phi \psi(x) e^{i(k_y y + k_z z)} e^{-iEt/\hbar} ,$$

where ϕ is a constant spinor and use the fact that if X is a matrix satisfying the relation $X^2 = \mathbf{1}$, then $\phi = (X \pm 1)\phi_0$ is an eigenvector of X .