

אוניברסיטת תל אביב

הפקולטה למדעים מדוייקים
ע"ש ריימונד וברלי סאקלר

סמסטר א', מועד א', תש"ס
תאריך הבחינה: 15.2.00

הבחינה: פיסיקה קוונטית 1 המורה: פרופ' בנימין סבטיצקי

משך הבחינה: שלש שעות.
אין להעזר ברשימות או בספרים.
ענו על שלוש מתוך ארבע השאלות. בהצלחה!

- N fermions with mass M are placed into a one-dimensional square well of infinite height, $0 < x < L$. All are in the same spin state.
 - Solve for the single-particle states $|n\rangle$.
 - Write the Hamiltonian H_0 for non-interacting particles in terms of creation and annihilation operators c_n, c_n^\dagger in the $|n\rangle$ basis.
 - Write the operator for the dipole moment $R \equiv \sum_{i=1}^N x_i$ in terms of c_n, c_n^\dagger . (Don't do any hard integrals. Call them $f(n, n')$, etc.)
- A photon with $\hat{\mathbf{k}} = \hat{\mathbf{z}}$ is in a polarization state characterized by Stokes parameters $(S_0, \mathbf{S}) = (1, 1, 0, 0)$.
 - Write the density matrix $\rho = \frac{1}{2}(S_0 + \mathbf{S} \cdot \boldsymbol{\sigma})$ explicitly.
 - Is the state pure or mixed?
 - Characterize the state as a mixture of pure states with probabilities p_i .
 - What are the Stokes parameters of the photon's state after it is passed through a polarizer aligned along $\hat{\mathbf{x}}$?
- Recall that

$$H_{\text{int}} = -\frac{e}{m} \mathbf{p} \cdot \mathbf{A}(\mathbf{r}) + \frac{e^2}{2m} \mathbf{A}(\mathbf{r})^2 - \frac{e}{2m} \boldsymbol{\sigma} \cdot \mathbf{B}(\mathbf{r})$$
$$\mathbf{A}(\mathbf{r}) = \frac{1}{L^{3/2}} \sum_{\mathbf{k}} \sum_{\lambda} \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} \left[\boldsymbol{\epsilon}_{\mathbf{k}\lambda} a_{\mathbf{k}\lambda} e^{i\mathbf{k}\cdot\mathbf{r}} + \boldsymbol{\epsilon}_{\mathbf{k}\lambda}^* a_{\mathbf{k}\lambda}^\dagger e^{-i\mathbf{k}\cdot\mathbf{r}} \right]$$

Write (but do not evaluate) the three atomic matrix elements that govern the fine-structure transition $2^2p_{3/2} \rightarrow 2^2p_{1/2}$ in hydrogen. Write them in the form $\langle \frac{3}{2} | \mathcal{O} | \frac{1}{2} \rangle$, where the operator \mathcal{O} is written explicitly, and indicate the multipolarity.

- Find a subgroup of the rotation group $\text{SO}(3)$ that has five elements. (Describe it in words.)
- Write a representation of the subgroup as a set of 3×3 unitary matrices.
- If your representation is reducible, show how to reduce it to smaller representations.