Running the Coupling

Douglas Ross

University of Southampton

Cuts not Poles

The Mellin transform of the BFKL amplitude involves an integral over ν.

$$f(\omega, \mathbf{k_1}, \mathbf{k_2}) \sim \int d\nu \left(\frac{\mathbf{k_1}^2}{\mathbf{k_2}}\right)^{i\nu} \frac{1}{\omega - \overline{\alpha_s}\chi(\nu)}$$

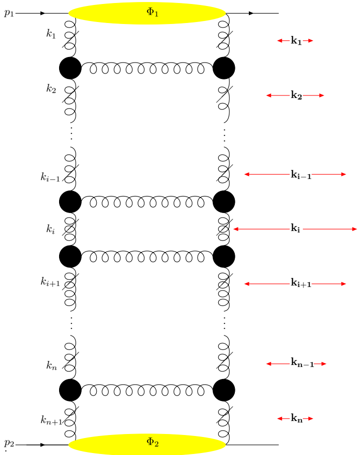
The singularity is a cut with branch-point

$$\overline{\alpha_s}\chi(0)$$

Regge theory predicts a pole !

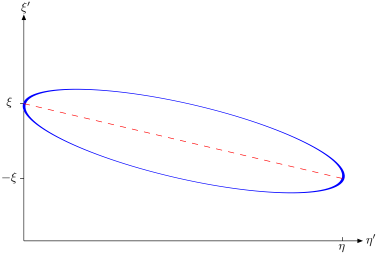
Diffusion

As we go down the BFKL ladder away from the impact factor the spread of $\mathbf{k}$ for which the integral has support, increases (until we start getting near the bottom of the ladder)



Bartels' Cigar

The BFKL equation is a diffusion equation in $\eta \sim \ln(s)$ and $\xi = \ln(\mathbf{k}^2/\Lambda^2_{QCD})$.



Eigenvalue equation

$$\mathcal{K}_0 e^{i\nu\xi} = \overline{\alpha_s}\chi(\nu)e^{i\nu\xi} = \overline{\alpha_s}\chi\left(-i\frac{\partial}{\partial\xi}\right)e^{i\nu\xi}$$

BFKL equation (as a diffusion equation)

$$\left[\frac{\partial}{\partial\eta} - \overline{\alpha_s}\chi\left(-i\frac{\partial}{\partial\xi}\right)\right] f(\eta,\xi) = \delta(\xi - \xi_0)$$

What value should be used for $\overline{\alpha_s}$?

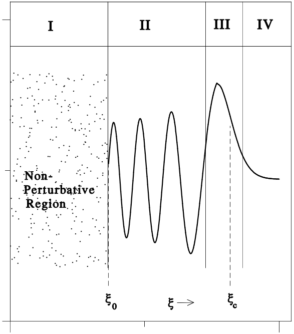
$$\overline{\alpha_s}(\xi) \sim \frac{C_A \beta_0}{\pi \xi}$$

Eigenfunction with eigenvalue ω

$$e^{i\nu\xi} \quad \left(\nu = \chi^{-1}\left(\frac{\omega \pi \xi}{\beta_0 C_A} \right) \right)$$

$$\xi_c = 4\ln(2)\frac{\beta_0 C_A}{\omega \pi}$$

[For $\xi > \xi_c$, ν is imaginary]



- Region II: $\xi \ll \xi_c$ Oscillating solution
- Region IV: $\xi \gg \xi_c$ Exponential decay
- Region III: ν is small

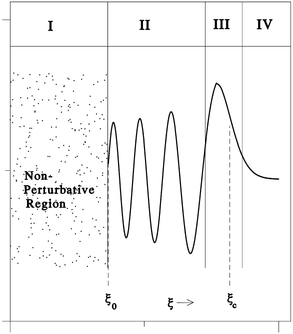
$$\omega - \chi_(\nu) \approx \frac{\omega}{\beta_0 C_A}(\xi - \xi_c)$$

Expanding χ to order ν^2

$$\left[\frac{\omega\pi}{\beta_0 C_A}(\xi - \xi_c) + \frac{\chi''(0)}{2}\frac{\partial^2}{\partial\xi^2}\right] f_\omega(\xi) = 0$$

Airy's equation.
Airy functions chosen to match functions and their derivatives in regions II and IV (fixes phase at II-III boundary).

- Region I: $\xi < \xi_0$, too small for perturbation theory to be reliable.



Now suppose the infrared behaviour in the non-perturbative region fixes the phase at the I-II boundary.

This means that only solutions with discrete values of ω can fit between regions I and II an obey the phase fixing at both boundaries

This gives separate poles, rather than a cut for the BFKL amplitude - consistent with the predictions of Regge theory.